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New Solar Attitude Control Approach for Satellites in Elliptic Orbits

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The investigation explores the feasibility of developing a simple solar attitude controller. The case of gravity-oriented satellites carrying two sets of solar surfaces in elliptic orbits has been considered. An approximate analytical approach has been used to evolve the suitable control criteria for regulating the movement of solar surfaces so as to counter the adverse effect of eccentricity normally responsible for a considerable deterioration in the attitude control characteristics of the conventional passive or semipassive methods. The solar controller thus developed is found to be quite effective in limiting the librational amplitude of motion. It is felt that the simplicity of the proposed controller configuration and ease of its operation make it particularly attractive for several satellite applications requiring modest attitude accuracies.

Nomenclature

A_j = total area of the set of control plates $S_j - S_j'$, $j=1,2$ (see Fig. 1)
C_j = solar parameter $= R_p^3 S A_j l_j (1+\rho_j)/(C'\mu I)$, $j=1,2$
C' = velocity of light
e = eccentricity of orbit
i = inclination of orbital plane with respect to the ecliptic
I $= I_{xx} = I_{yy} > I_{zz}$
I_{xx}, I_{yy}, I_{zz} = moments of inertia about x,y,z axes, respectively
K_i = inertia parameter $= 1 - I_{zz}/I$
l_j = distance of geometric center of the set of control plates $S_j - S_j'$ from the satellite mass center, $j=1,2$ (see Fig. 1)
p_j $= \frac{1}{2}(1-\rho_j)/(1+\rho_j)$, $j=1,2$
$Q_\psi, Q_\beta, Q_\lambda$ = generalized forces due to solar radiation pressure
R = distance between mass center and center of force (see Fig. 1)
R_p = perigee distance
S = solar constant
t = time
x,y,z = principal body coordinates with origin at mass center s (see Fig. 1)
β_1 $= -\sin\phi(1-\cos i)$
β_2 $= -\sin\phi \sin i$
θ = satellite position angle, as measured from the perigee (see Fig. 1)
μ = gravitational field constant
ρ_j = reflectivity of the control surfaces $S_j - S_j'$, $j=1,2$ (see Fig. 1)
ϕ = solar position angle
$\Delta\phi$ = maximum permissible deviation in ϕ before permitting the resetting of the control surfaces
ϕ_c = reference solar position angle used for initial setting of the control surface
ψ, β, λ = librational angles denoting pitch, roll, and yaw degrees of freedom
$(\ )'$ $= d(\ )/d\theta$
$(\ )_0$ = initial conditions

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Introduction

THE possibility of using solar radiation pressure (SRP) for attitude control has been explored by several investigators.[1-6] Even though the effectiveness of this concept has been clearly established, particularly for satellites in high-altitude orbits, it appears to have found only limited acceptance by the spacecraft designers. This lack of enthusiasm for the solar attitude controllers may be partly attributed to the availability of the well-proven conventional passive and semipassive alternatives using gravity gradient, satellite spin, magnetic torquing, etc. The complexity of the proposed controller models and their operation has perhaps been a major detrimental factor as well. It seems that from the very beginning, there has been undue emphasis on using the combination of proportional and derivative feedback control policies for acheiving a high degree of pointing accuracy. In the process, certain obvious possibilities emphasizing the simplicity of control and operation appear to have been overlooked.

This paper investigates the feasibility of developing a simple solar attitude controller for satellites operating in

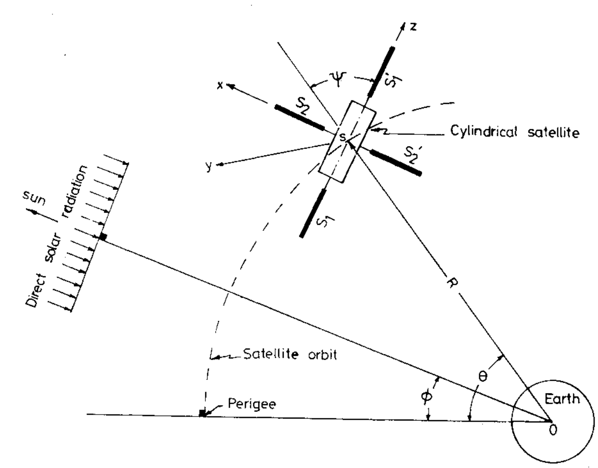

Fig. 1 Geometry of satellite motion.

elliptic orbits. Attention is focused on evolving a simple control strategy which can counter the adverse effect of eccentricity normally responsible for worsening of the attitude characteristics of the conventional methods.

Formulation of the Problem

The formulation begins with the analysis of motion of a cylindrical satellite in elliptic orbit, in an arbitrary plane about the earth-center 0 (Fig. 1). The satellite model considered here carries two sets of plates made of light, rigid, but highly reflective material (e.g., aluminized mylar membrane). The two sets are assumed to be mounted with their surfaces normal to the orbital plane. Denoted by x,y,z are the principal coordinate axes of the satellite with origin at the mass-center s. Using the Lagrangian formulation, the governing equations for pitch ψ, roll β, and yaw λ librations can be written as

$$
\begin{aligned}
&[(1+e\cos\theta)\psi'' - 2e(1+\psi')\sin\theta] \\
&\quad -2(1+e\cos\theta)(1+\psi')\beta'\tan\beta + 3K_i\sin\psi\cos\psi \\
&\quad -(1-K_i)\{(1+e\cos\theta[\lambda'' - (1+\psi')\beta'\cos\beta - \psi''\sin\beta] \\
&\quad -2e\sin\theta[\lambda' - (1+\psi')\sin\beta]\}(\sin\beta/\cos^2\beta) - (1-K_i)\beta' \\
&\quad \times\{\lambda' - (1+\psi')\sin\beta\}/\cos\beta = Q_\psi
\end{aligned}
\tag{1a}
$$

$$
\begin{aligned}
&[(1+e\cos\theta)\beta'' - 2e\beta'\sin\theta] \\
&\quad +[(1+e\cos\theta)(1+\psi')^2 + 3K_i\cos^2\psi]\sin\beta\cos\beta \\
&\quad +(1-K_i)(1+e\cos\theta)(1+\psi') \\
&\quad \times[\lambda' - (1+\psi')\sin\beta]\cos\beta = Q_\beta
\end{aligned}
\tag{1b}
$$

$$
\begin{aligned}
&(1+e\cos\theta)[\lambda'' - (1+\psi')\beta'\cos\beta - \psi''\sin\beta] \\
&\quad -2e[\lambda' - (1+\psi')\sin\beta]\sin\theta = Q_\lambda
\end{aligned}
\tag{1c}
$$

where $Q_i\,(i=\psi,\beta,\lambda)$ represent the generalized forces due to the SRP. Without going into the details of their evaluation which can be accomplished through the principle of virtual work, it can be shown that for specular reflection the expressions for the generalized forces can be written as[6]

$$
\begin{aligned}
Q_\psi = {} & [(1+e)/(1+e\cos\theta)]^3(1/\cos\beta)\{C_1[\sin(\theta+\psi-\phi) \\
& +\beta_1\cos(\theta+\psi)]\ |[\sin(\theta+\psi-\phi)+\beta_1\cos(\theta+\psi)]| \\
& -C_2[\cos(\theta+\psi-\phi)-\beta_1\sin(\theta+\psi)]\ |[\cos(\theta+\psi-\phi) \\
& -\beta_1\sin(\theta+\psi)]|\ \}
\end{aligned}
\tag{2a}
$$

$$
\begin{aligned}
Q_\beta = {} & 2[(1+e)/(1+e\cos\theta)]^3\{p_1C_1[\sin\beta\cos(\theta+\psi-\phi) \\
& -\beta_1\sin\beta\sin(\theta+\psi)-\beta_2\cos\beta]\ |[\sin(\theta+\psi-\phi) \\
& +\beta_1\cos(\theta+\psi)]|\ +p_2C_2[\sin\beta\sin(\theta+\psi-\phi) \\
& +\beta_1\sin\beta\cos(\theta+\psi)-\beta_2\cos\beta] \\
& \times\ |[\cos(\theta+\psi-\phi)-\beta_1\sin(\theta+\psi)]|\ \}
\end{aligned}
\tag{2b}
$$

$$
Q_\lambda = 0 \tag{2c}
$$

It is interesting to note that even for satellites in an arbitrary nonecliptic orbit, the solar torque corresponding to the yaw mode is always zero. Furthermore, in the situation of highly reflective control surfaces, the roll torque also vanishes almost completely. It is thus apparent that the solar torque, by itself, cannot excite the roll or yaw modes of librational motion when the solar surfaces are highly reflective as proposed for the control model. On the other hand, in the extreme situation of totally absorptive control surfaces for satellites in nonecliptic orbits, the SRP introduces disturbing roll moment as well as some modifications in the pitching torque. However, an order-of-magnitude analysis of the various solar terms shows that even the moderately large

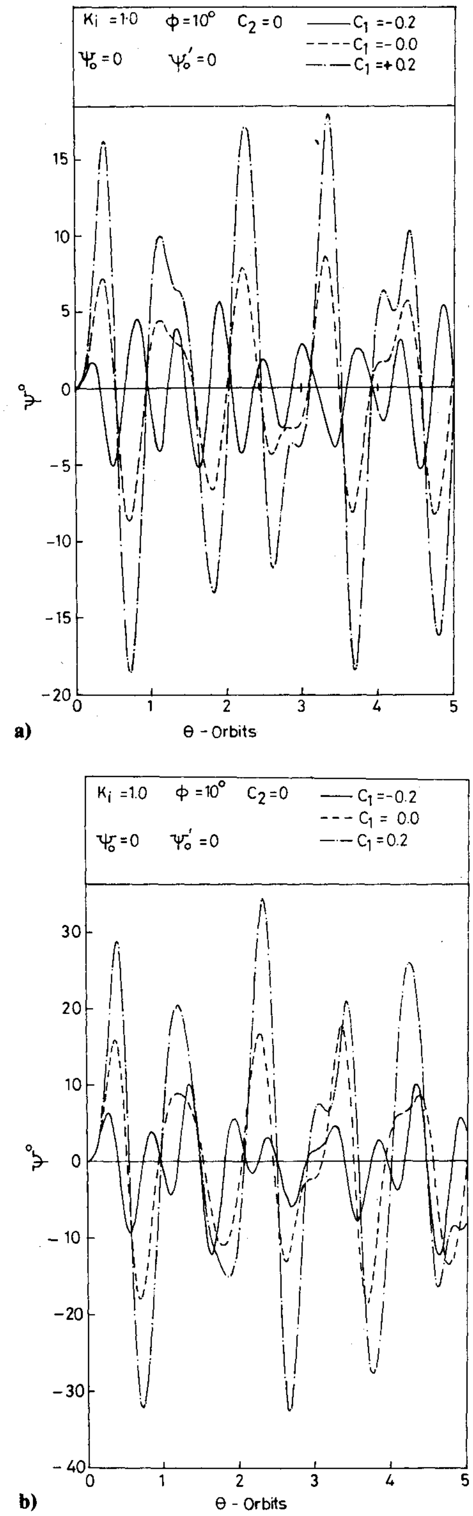

Fig. 2 Effect of solar radiation pressure on librational response for a) $e=0.1$ and b) $e=0.2$.

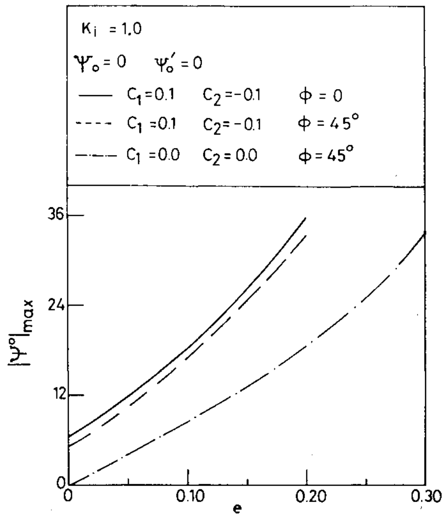

Fig. 3 System plots showing the effect of eccentricity on maximum librational amplitude in the presence of solar radiation pressure.

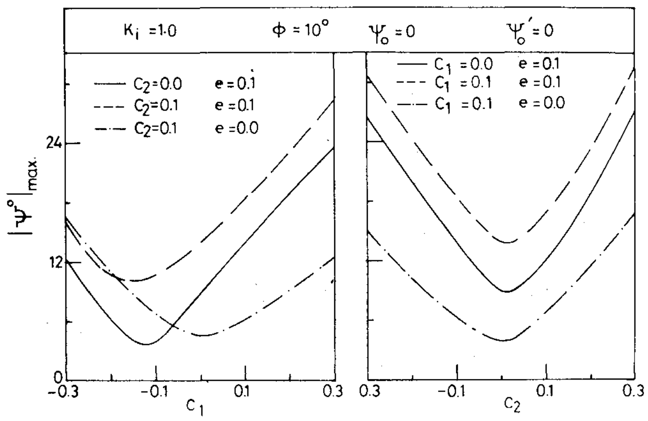

Fig. 4 System plots showing the effect of solar parameters on maximum librational amplitude.

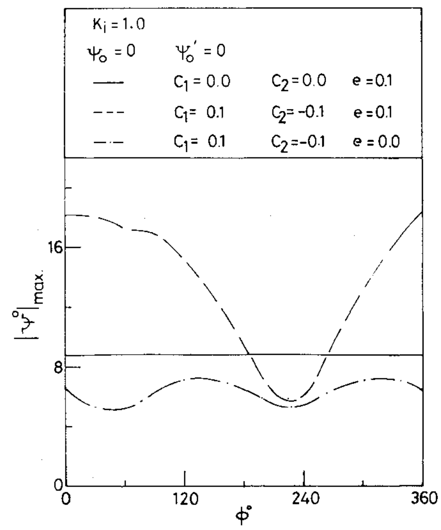

Fig. 5 System plots showing the effect of solar position angle on maximum librational amplitude.

orbital inclinations of up to $\sim \pm 30$ deg have but only a second-order influence on the generalized forces. It is thus evident that an investigation of the much simpler, particular case of planar librations of the satellites moving in the ecliptic plane would be of considerable significance. Attention was therefore focused on the planar analysis involving the satellite pitching librations governed by the differential equation which now takes the simplified form

$$(1+e\cos\theta)\psi'' - 2e(1+\psi')\sin\theta + 3K_i\sin\psi\cos\psi$$

$$= [(1+e)/(1+e\cos\theta)]^3 [C_1\sin(\theta+\psi-\phi)\ |\sin(\theta+\psi-\phi)|$$

$$-C_2\cos(\theta+\psi-\phi)\ |\cos(\theta+\psi-\phi)|\ ] \qquad (3)$$

where C_1, C_2, referred to as the solar parameters in the analysis, are given by

$$C_j = l_j A_j (1+\rho_j) R_p^3 S/(\mu I C') \quad (j=1,2) \qquad (4)$$

Effect of Solar Radiation Pressure on Attitude Dynamics

The governing nonlinear nonautonomous differential equation (3) does not have a closed-form solution. Numerical techniques were therefore used to evaluate librational performance of the satellites in various practical situations. Numerical integration was performed using Adam's extrapolation method[7] in conjunction with a suitable stepsize of 2 deg.

Figure 2 shows the librational response in two typical situations with different orbital eccentricities. To bring out the influence of the SRP clearly, the response for $C_1 = C_2 = 0$, i.e., when the solar effect is ignored, has also been included for comparison. It may be observed that the SRP increases the amplitude of librations when C_1 is positive. On the other hand, when negative values are chosen for this parameter, the SRP seems to influence the system favorably, reducing the maximum amplitude of librational motion.

To understand the dynamic behavior of satellites over a wide range of system parameters, numerous response plots were obtained. The resulting information is condensed in the form of system plots (Figs. 3-5).

Figure 3 shows the effect of eccentricity on the librational amplitude in three different situations. It may be observed that, in general, the amplitude continually increases with increase in eccentricity. Here, solar parameters C_1, C_2, and the solar position angle ϕ appear to be the other significant parameters of the system affecting the librational motion.

Figure 4 indicates the effect of the solar parameters C_1, C_2 on the librational response. It may be pointed out that in case of circular orbits and for $C_1 = C_2 = 0$, the satellites with their long axes initially oriented along the local vertical do not execute the librational motion unless disturbed. In contrast, the response results for any other nonzero combination of the solar parameters exhibit considerable amplitude of librations, thus establishing the detrimental influence of the SRP in the circular orbits. However, it is interesting to note that for elliptic orbits, the minimum amplitude does not correspond to the case $C_1 = C_2 = 0$, i.e., when the solar effect is ignored. Furthermore, through a judicious choice of the solar parameters, it is possible to reduce the librational amplitude substantially. The significance of this result cannot be overemphasized, as it suggests an attractive possibility of countering the adverse influence of eccentricity on attitude motion of satellites.

The effect of the varying solar position angle is indicated in Fig. 5. It may be noted that the solar position has a considerable influence on the librational amplitude of satellites, particularly in elliptic orbits. This suggests that a change in ϕ significantly affects the choice of the solar parameters for achieving an improvement in the satellite librational performance.

Thus, the numerical results presented here clearly establish the possibility of achieving considerable reduction of the maximum librational amplitude through an appropriate choice of the solar parameters. This beneficial aspect of the SRP appears to have been either missed or entirely ignored so

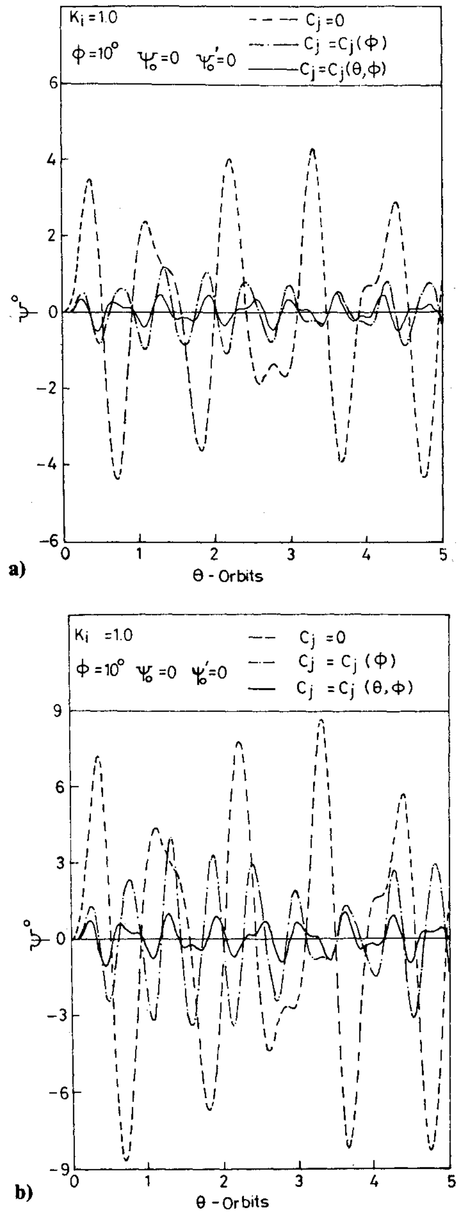

Fig. 6 Typical satellite response showing the attitude control through the proposed solar controller for a) $e = 0.05$ and b) $e = 0.1$.

far. On the other hand, an impression seems to have gained ground that the uncontrolled solar effect is always detrimental to the attitude performance. This may perhaps partly explain why, in the recent developments of the various solar controller models, the emphasis has been essentially on regulating the solar torque through the moving surfaces using complex feedback control systems.

The system plots obtained here can no doubt be used for selecting the appropriate values of the solar parameters in several cases. However, to numerically generate the extensive data to enable the choice of these parameters covering all possible situations is neither feasible nor desirable. So, it is now proposed to treat the problem analytically with a view to developing the suitable criteria facilitating a judicious choice of the solar parameters.

Synthesis of Control Strategy Using Approximate Analytical Approach

It was felt that in view of the complexity introduced by the nonlinear and nonautonomous nature of the problem, it would be essential to make some reasonable assumptions for treating the equations of motion analytically. It is easy to see

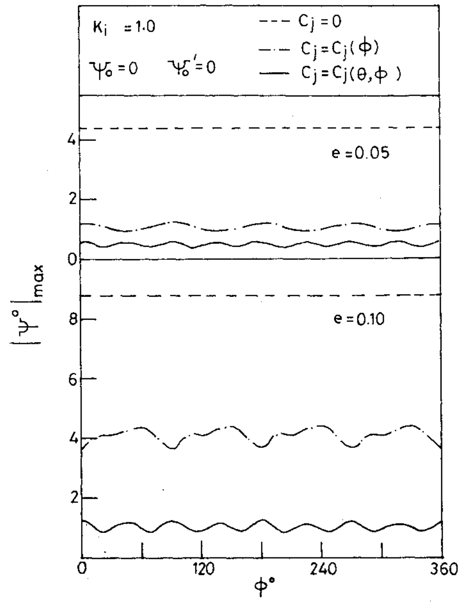

Fig. 7 System plots showing the attitude control characteristics as affected by the solar position angle.

that for the practical situation of small-amplitude librations assumed here, the governing equation can be written as[8]

$$(1+e\cos\theta)\psi'' - 2e\psi'\sin\theta + 3K_i\sin\psi\cos\psi$$
$$= 2e\sin\theta + [(1+e)/(1+e\cos\theta)]^3 [C_1\sin(\theta-\phi)$$
$$\times |\sin(\theta-\phi)| - C_2\cos(\theta-\phi)\ |\cos(\theta-\phi)|\ ] \tag{5}$$

It may be pointed out that in general the deterioration of the attitude control performance characteristics is mainly due to the periodic pitching excitation represented by $2e\sin\theta$. An attempt was therefore made to explore the possibility of getting rid of this term with the help of the last two terms representing the solar pressure effects. To achieve this objective, the restructuring of the solar terms was considered so that, with a suitable choice of C_1, C_2, the right-hand side in the equation could be forced to disappear. Fortunately, Fourier expansion of the terms appearing within the square brackets in Eq. (5) suggests that they can be effectively described by their first harmonic.[8] On incorporating this approximation, Eq. (3) simplifies to

$$(1+e\cos\theta)\psi'' - 2e\psi'\sin\theta + 3K_i\sin\psi\cos\psi$$
$$= 2e\sin\theta + (8/3\pi)[(1+e)/(1+e\cos\theta)]^3$$
$$\times [C_1\sin(\theta-\phi) - C_2\cos(\theta-\phi)] \tag{6}$$

On separately grouping the terms involving $\sin\theta$ and $\cos\theta$, it was found that the right-hand side in Eq. (5) vanishes when

$$C_1 = -0.75\pi[e/(1+e)^3](1+e\cos\theta)^3\cos\phi \tag{7a}$$

$$C_2 = 0.75\pi[e/(1+e)^3](1+e\cos\theta)^3\sin\phi \tag{7b}$$

These results are of considerable significance, as they clearly bring out the effects of the orbital eccentricity as well as the sun position angle on the desired instantaneous values of the solar parameters. It may be pointed out that by suitably regulating the movement of the pairs of the solar surfaces $S_1 - S_1'$ and $S_2 - S_2'$, respectively, along the z and x axes (Fig. 1), l_1, l_2, and hence C_1, C_2, can be varied as desired. Thus, using these relations, it may be possible to control the solar parameters suitably in order to overcome the adverse effect of eccentricity.

A careful examination of the control relations reveals that the changing satellite position in orbit is likely to be of little consequence to the selection of C_1, C_2, particularly for low values of orbital eccentricities. Hence, these relations can be considerably simplified:

$$C_1 = -0.75\pi[e/(1+e)^3]\cos\phi \tag{8a}$$

$$C_2 = 0.75\pi[e/(1+e)^3]\sin\phi \tag{8b}$$

To examine the validity of the approximate analytical approach, the preceding expressions developed for C_1, C_2 were substituted in Eq. (3), and the resulting equation of librational motion was integrated numerically. Some typical response results thus obtained are shown in Fig. 6. It may be observed that the control schemes developed analytically are quite effective in limiting the amplitude of librations to much smaller values. Even with the highly simplified control of solar parameters represented by Eq. (8) where $C_1 = C_1(\phi)$, $C_2 = C_2(\phi)$, the amplitude reduction is found to be considerable. The more stringent control scheme represented by Eq. (7), where $C_1 = C_1(\theta,\phi)$, $C_2 = C_2(\theta,\phi)$ further improves the controller effectiveness.

To assess the influence of the important system parameters on controller effectiveness, the response results were analyzed in some detail. Figure 7 shows the effect of the solar position angle on the system behavior. As the position of the sun changes continuously in the ecliptic plane, this study would be particularly useful in predicting the long-range satellite librational dynamics. The analysis shows that the changing position of the sun does not significantly alter the attitude control characteristics, particularly for low values of eccentricities.

Figure 8 exhibits the effect of orbital eccentricity on maximum librational amplitude. It may be observed that both control strategies are quite effective in reducing the amplitude of motion. For continuously increasing values of eccentricities, the effectiveness of the control schemes progressively declines. The degradation in performance is particularly severe with the simpler control scheme. This limits the applicability of the ϕ-sensitive control to only slightly eccentric orbits. However, the usefulness of the proposed control scheme, sensitive to both θ and ϕ, extends to a much larger range of eccentricities. It is interesting to note that even for an eccentricity as large as 0.3, it can limit the amplitude to well within 5 deg. In view of the recently proposed use of highly elliptic synchronous orbits in order to maximize the net on-station weight capability,[9] this study may be particularly significant.

Attention is now focused on the comparative assessment of the overall attitude performance characteristics associated with the two proposed control laws for the particular case of satellites in near-circular orbits. It was felt that, in this situation, even though the control operation sensitive to both θ and ϕ provides slightly better pointing accuracy, the ϕ-based action would result in considerably lower demand on the control system. It may be pointed out that the simple, ϕ-sensitive control involving the single variable is easier to realize than the other one, dependent on both θ and ϕ. Furthermore, as the period of terms such as $\cos\phi$ and $\sin\phi$ is one year—much higher than the orbital period of the θ terms involved in the control relations—it is apparent that another major advantage of the simple ϕ-sensitive control function lies in its much lower frequency. It may be possible to use this fact to advantage by effectively replacing the continuous periodic control action by a limited number of prespecified discrete operations over each cycle. To assess the effectiveness of this approach, the response plots were obtained for the varying degrees of discretization of the proposed ϕ-sensitive control criterion. The results of this analysis are shown in Fig. 9. Here, ϕ_c represents the solar position angle with respect to which the solar control surfaces are assumed to be set initially and $\Delta\phi$ denotes the maximum permissibile deviation in ϕ before permitting the resetting of the solar surfaces. It may be observed that the discretization leads to considerable deterioration in the attitude control characteristics. However, for values of $\Delta\phi$ representing small changes in ϕ of, say, up to ± 5 deg, the minor degradation in librational performance is likely to be well within the tolerance limits for many space applications. This result is of considerable practical significance, as it establishes the feasibility of employing the solar surfaces suitably mounted on satellites to achieve substantial reduction in librational amplitude, even without exercising control during the long intervals extending up to as much as a week. The proposed control approach thus suggests a simple method for testing the concept of achieving librational control using the SRP with minimal changes in satellite design.

The proposed controller presents an interesting possibility of controlling satellite orientation in elliptic orbits. The realization of the control is easy and the size of the plates required quite modest. A preliminary estimate using the data of the INTELSAT IV series of satellites shows that the plate size of 0.2 m^2 with permissible movement of 20 cm is adequate for orbital eccentricities of 0.1. For larger eccentricities, this requirement increases in nearly the same proportion. The semipassive character of the system promises increased lifetime with overall reduction in the cost.

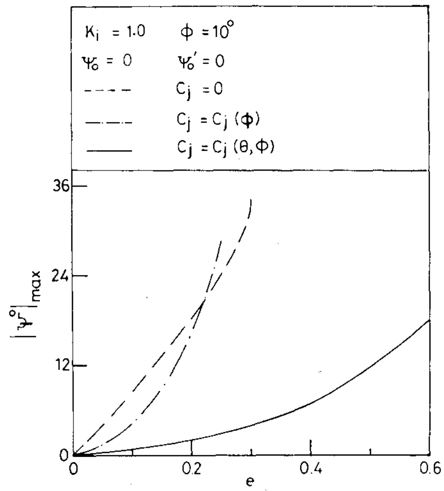

Fig. 8 System plots showing attitude control characteristics as affected by the eccentricity.

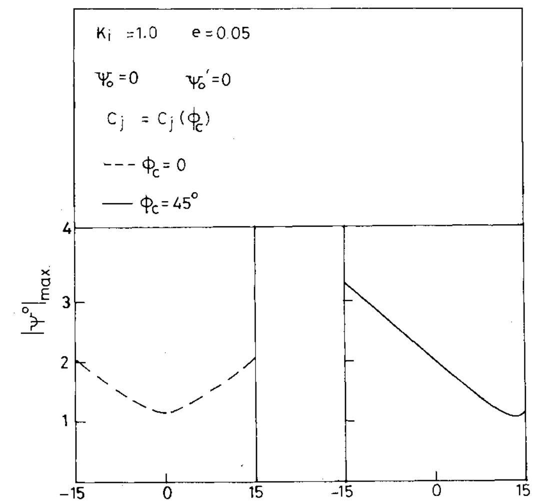

Fig. 9 System plots showing the effect of changing the solar position angle from its nominal value.

Concluding Remarks

The important features of the analysis and the conclusion based on them may be summarized as follows:

1)Contrary to what is generally believed, the SRP need not be always detrimental to the satellite attitude response. In fact, there are several situations where, for a proper choice of the solar parameters, the radiation pressure can substantially reduce the maximum librational amplitude of satellites in elliptic orbits.

2)The system plots, developed numerically, summarize the satellite librational response characteristics over a wide range of parameters. Through these plots, it is possible to select the appropriate values of the solar parameters minimizing the librational amplitude in several cases.

3) The approximate analytical approach adopted here seeks to make use of the solar terms in the equation of librational motion to eliminate the major excitation effect of eccentricity normally responsible for the worsening of the attitude control characteristics of the conventional methods. The two control strategies thus evolved are found to be quite effective in restraining the satellite oscillations.

4) The proposed control schemes are much simpler, as they are dependent only on θ and ϕ which, in turn, can be expressed as a function of time alone. This makes them particularly suited for preprogramming of the control function—unlike the earlier complex feedback control approaches which are sensitive to the librational disturbances and require on-board sensing and computation of the error signal for actuating the controls.

5) Using the relatively more stringent control law governed by the instantaneous values of the position of the sun (ϕ) and satellite (θ) leads to drastic reduction in maximum librational amplitude. It considerably extends the effective range of possible eccentricities for stable satellite operation. It is interesting to find that even for eccentricity as large as 0.3, the proposed controller can limit the maximum amplitude to well within 5 deg. This suggests the possibility of using it as an auxiliary control device for improving the overall librational performance characteristics of satellites in highly elliptic orbits. The proposed control mechanism may thus have far-reaching implications. In view of its ability to overcome the adverse effect of eccentricity to a large excent, it may lead to greater acceptance of highly elliptic orbits, particularly for multi-space missions.

6) The simpler ϕ-sensitive control strategy is quite effective for satellites in slightly eccentric orbits. Furthermore, it is possible to effectively replace this continuous periodic control action by a limited number of prespecified discrete operations over each cycle. The analysis establishes the feasibility of employing the suitably mounted solar surfaces on satellites to improve the librational performance even without exercising control for long intervals. The proposed control approach thus suggests a simple method for testing the concept of librational control using the SRP with minimal changes in satellite design.

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